

# APPLICATION OF THE CHAOS THEORY IN STUDYING THE PROCESS OF CONTINUAL FORMATION OF THE HUMAN RESOURCES

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**ABSTRACT:** This study represents an application of the mathematical theory provided by “The Chaos Theory” in the modelling and simulation of the continuous formation of human resources. Modelling will be done by using the time series and the R/S analysis of Benoit Mandelbrot. The simulation model is carried out using the SPSS 16 software on a series of data targeting the analyzed level or, in perspective, the knowledge, skills, attitudes, competencies and human behaviours. This study concludes that the process of continual human resources formation in organizations is self-correlated and depends on the previously recorded conditions.

## 1. INTRODUCTION

The Chaos Theory is a discipline belonging to modern mathematics and physics which describes the behaviour of non-linear dynamic systems. The common feature of these systems is the instability called „sensitivity” over the initial conditions. For this reason, the behaviour of non-linear dynamic systems targeting various time lengths is unpredictable and seemingly chaotic. Hence, it is named “The Chaos Theory.”

The “sensitivity” to the initial conditions is expressed through the infinitesimal differences of the initial conditions of systems which could generate substantially different outcomes in the evolution of the same systems. Non-linear systems evolve by being structured in fractal forms, where one and the same form can be recognized at different scales, and/or by repeating itself almost identically during its evolution.

Perceptually, nature works “non-linear” and seemingly chaotic. The situation has led to various opinions relating the Chaos Theory to disorder. However, this is a misconception when in fact the term „chaos” in the Chaos Theory refers to the inherited order of the system.

The systems described by the Chaos Theory are apparently disordered; nonetheless, this chaos is nothing else than a state of reorganization of a system [1]. In fact, by applying the mathematical framework provided by the Chaos Theory we target the identification of the internal order and the balance of the non-linear systems in their evolution. The main idea behind the Chaos Theory is the identification in the environment of the dynamic systems of (a) contradictory terms and (b) strain generated by contradictions, such as: learning and forgetting, accumulation and relaxation and so on.

Non-linear systems have a behaviour that can be described only (a) by the interaction between its components and (b) by summing up its qualities [4].

Due to the interaction between its components, the evolution in time of the non-linear systems could be modelled through non-linear differential equations and the R/S analysis. The latter is used in this study.

Following the improved computing power, the Chaos Theory has been applied successfully in many fields. From this prospective, this article shall attempt to apply the Chaos Theory to the field of the continual formation of the human resources.

## 2. R/S ANALYSIS

The need of continual formation of the human resources, the strategies established for its implementation and the operational activities related to preparing, organizing, deploying and implementing the training programs, could be considered as being dynamic systems whose relatively small initial conditions’ variation could cause significant differences in the evolution of the processes characteristic to the continual formation of the human resources. These sensitivities will be associated to other potential events, situations and circumstances which could disrupt the policies and training activities initially set, impacting the outcome of training objectives, thus generating a state of apparent chaos.

Under these conditions, the process of continual formation of the human resources can be mathematically modelled by using the Chaos Theory, respectively the Benoit Mandelbrot’s R/S analysis [4]. The R/S analysis could be applied to any set of data variable in time. In addition, it is also a dependency verification method of the subsequent outcome to the precedent one.

The established relationship proves the existence of a dependency of the current level to the previous level of the human resources formation. It is considered to be the most robust method of analyzing the non-Gaussian data and was depicted by Mandelbrot, de Hurst, Feder and recently Peters [4].

For applying this method of analysis to our process, we assign numerical values from 1 to 5 on a Likert Scale to the level of knowledge, skills, attitudes, competencies and human behaviours recorded at a certain time.

The basis of the R/S analysis in the human resources context is the adaptation of the principles set by Mandelbrot, as follows:

**Principle 1:** The values analyzed at one time may or may not depend on the trends of previous ones, which means that a certain value of the used data may be independent or not on the previously recorded values.

In applying the R/S analysis, the values of specific variables used in the process of continual formation of the human resources, such as: skills, knowledge, competencies, attitudes and behaviours are numerical and assessed subjectively. The level of these variables can be influenced by the previous trends and the level of certain variables may or may not be dependent on the level of the previous ones.

**Principle 2:** The subsequent trends of the analyzed values vary in time and could register various values compared to the values of the initial trends.

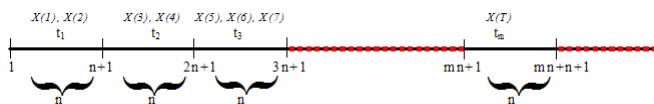
In the dynamic of the continual formation of the human resources, the effects over time of these trends will generate various variations from the initial conditions. „Sensitivity” (variation) over the initial conditions, means that even the smallest changes of the initial conditions of this process may lead to radical differences in its development.

Basing the R/S analysis on these principles ensures the achievement of its purpose, namely, the collection of meaningful information regarding the „memory” of the time series in the evolution of the process of continual human resources formation and the predictability of the encountered values.

The analysis uses the following data:

1. Time-scale index „n”

In order to examine how the specific variables of the continual human resources formation process could be influenced by the previous trends, the time index „n” corresponding to the time scale parameter “T” will be used, where “T” is the sum of the time intervals “n”. The length of time scale consists of several segments of length “n”, as provided in Figure 1:



**Figure 1.** Time scale for the period “T”.

where:

- $t_1, t_2, t_3, \dots, t_m$  stand for „m” periods of time, each having the length „n”;
- $(1; n+1); (n+1, 2n+1); (2n+1, 3n+1); \dots (mn+1, mn+n+1)$  are intervals of length “n” of the time scale, delimited by the points of division highlighted in Figure 1.

For each interval of length “n”, the  $X(1), X(2), X(3), \dots, X(T)$  data represent the values of the level of continual human resources formation, such as: skills, behaviours, and so on, and are bound to adjacent intervals of length „n”. The R/S analysis involves the calculation of each of these intervals’ average, while the index „n” represents the average level of continual human resources formation on the time scale that has the length „n”.

Example:

On the time scale “T”, if the first interval of length „n” includes the values of the variables  $X(1), X(2)$ , the R/S analysis generates the average of the values included in this interval, respectively

$$[X(1)+X(2)] / 2 = M_1$$

Thus, if the values are:  $X(1) = 3$  and  $X(2) = 5$ , then the arithmetic average  $M_1$  is 4. This represents the average level of the continual human resources formation in the first period of the time scale having the index „n.” Nonetheless, in the following interval of length „n” and the next adjacent intervals, the average value of the index „n” is specific to each period:  $M_1, M_2, \dots, M_m$ .

The index „n” provides to the human resources managers information about the memory of the process. Further, it could be used to highlight the memory of the system for periods of

time having the length „n” and contributes to predicting the values that the level of the human resources formation will record.

2. Range “R”

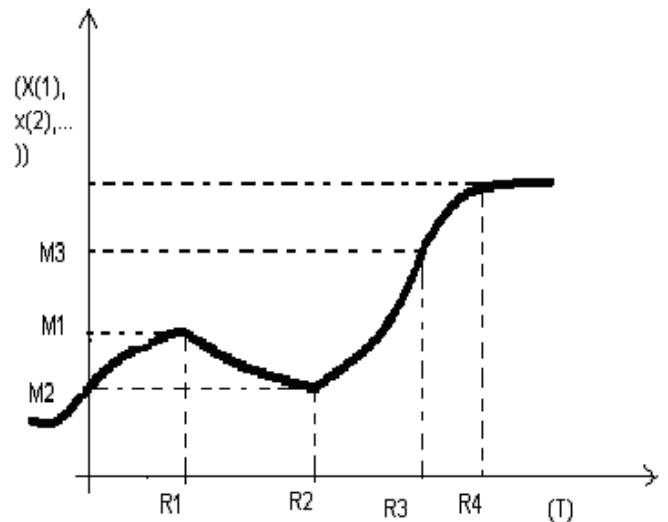
The interval is defined as the difference between the maximum and the minimum of the  $X(1), X(2), X(3), \dots, X(T)$  values displayed on the Figure 1. Consequently, this represents the distance between two extreme values within the same interval of length „n” for a system whose time intervals vary:  $R = X_{max}(t) - X_{min}(t)$

Example:

The R value is generated by considering the values of the variables being  $X(1) = 3$  and  $X(2) = 5$  from the first interval and applying them into the above formula, consequently

$$R = 5-3 = 2.$$

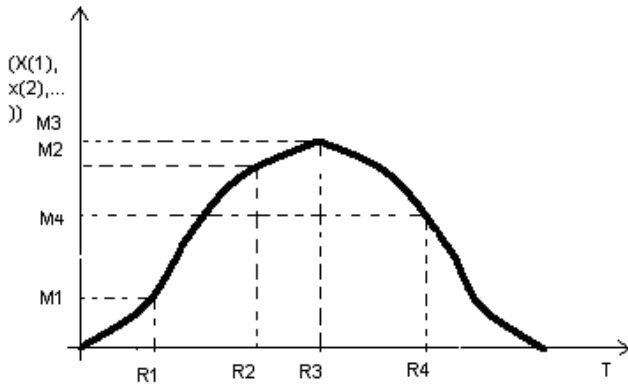
By using the two measuring instruments mentioned at points 1 and 2, the distribution of the continual human resources formation values will resemble a non-Gaussian probabilistic distribution, see Figure 2. The OX axis (time series „T”) displays the  $R_1, R_2, R_3,$  and  $R_4$  values („range” R), while the OY axis displays the values of the  $M_1, M_2, M_3$  and  $M_4$  averages, corresponding to the levels of continual human resources formation.



**Figure 2.** The non-Gaussian distribution curve of the values of the level of continual human resources formation.

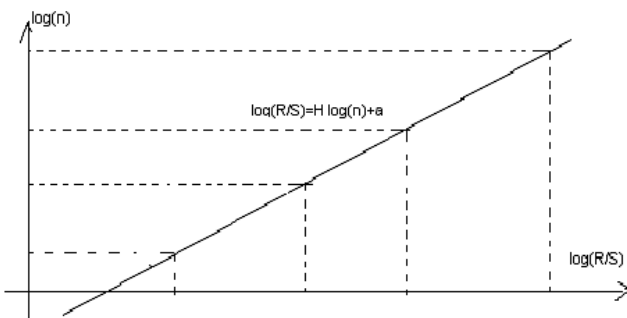
3. Rescaled range “S”

The rescaled range indicator is used to arrange the data series analyzed above on a Gaussian type curve, see Figure 3. This range is obtained by dividing the time interval „T” into intervals equal to the standard deviation of the data series  $\{X(1), X(2), X(3), \dots, X(T)\}$ . Further, the standard deviation is calculated relative to the average value  $M_1, M_2, \dots, M_m$  of the continual human resources formation during the entire period “T”. In this case, the time interval „n” is equal to the standard deviation of the data series relative to the statistical average of  $M_1, M_2, \dots, M_m$ . Without this normalization it would be impossible to compare the different origins of the time series that generate the trends of the continual human resources formation.



**Figure 3.** Gaussian distribution curve of the average values of the continual human resources formation.

The R/S analysis was created by Hurst to compare various dynamic phenomena and processes. The two principles above detailed are the cornerstone of the analytical approach which led to the R/S analysis. The Hurst statistics shows that the R/S ratio depends on the factor “H” by the means of an exponential law, such as:  $R/S \sim n^H$  [2,5]. Applying a logarithm to this exponential law, the dependency relationship between the logarithm of the R/S and the logarithm of the time scale index „n” is obtained. Through this process, the Hurst exponent becomes the slope of the regression line formed by the pairs of points experimentally determined:  $(\log(n), \log(R/S(n)))$ , as shown in Figure 4.



**Figure 4.** The dependency of the ratio R/S on the time scale of the values of continual human resources formation.

Hurst's exponent, „H”, is the ratio between  $[\log(R/S) - a]$  and  $\log(n)$  and is the fractal dimension of the time series used in estimating the continual human resources formation.

Depending on the value of the Hurst exponent, that may be higher or lower than 0.5, the series estimating the level of continual human resources formation could either be persistent or anti-persistent, meaning that it could increase in a certain period and decrease in the following period, or vice-versa. Using Hurst statistic, the future value is related to the present value using the correlation coefficient C given by  $C = 2^{2H-1} - 1$ , see [3, 6]

The classification of the persistent or anti-persistent series is based on the sign of the exponent  $2H-1$  in the above formula where:

if  $2H-1 \geq 0$ , then  $2H \geq 1$ , which means that  $H \geq 1/2 = 0.5$ .

### 2.1. Conclusions of R/S Analysis

Depending on the value of the Hurst exponent, “H”, the values of the variables of the continual human resources formation are either self-correlated or not, relative to the time series X(t), as follows:

a.)  $H = 0.5$ , defines a time series X (t) where there is no correlation of the values of the variables of the continual human resources formation;

b.)  $H > 0.5$ , defines a time series X (t) where there is a trend of increasing the values of the variables of the continual human resources formation during a previous period of time. This ascending trend will be preserved in any other future period of time;

c.)  $H < 0.5$ , is characteristic to the self-regulated time series X(t). In this case, an ascending trend is immediately succeeded by a decreasing trend and vice-versa.

## 3. SIMULATION WITH THE SPSS 16 SOFTWARE PROGRAM

**Step 1.** Using the SPSS 16 software, the serie-timp.sav file was created with the time series  $\{X(1), X(2), X(3), \dots, X(T)\}$ . A 1 to 5 scale was used for assessing the level of continual human resources formation, see Figure 5:

Name	Type	Width	Decimals	Label	Values	Missing	Columns	Align	Measure
NIVELFRU	Numeric	8	2	Nivelul de formare a resurselor umane estimat pe o scală 1-5	None	None	8	Right	Scale
NIVELF_1	Numeric	9	2	DIFF(NIVELFRU,1)	None	None	11	Right	Scale
NIVELF_2	Numeric	9	2	DIFF(NIVELFRU,1)	None	None	11	Right	Scale

**Figure 5.** Step 1 - SPSS 16 data entry.

**Step 2.** As detailed in the Figure 6, the data  $\{X(1), X(2), X(3), \dots, X(T)\}$  representing a subjective assessment of the numerical values corresponding the level of the continual human resources formation was added to the serie-timp.sav file.

	NIVELFRU	NIVELF_1	NIVELF_2	Y\$T	Y\$F
1	1,00	.	.		
2	2,00	1,00	1,00		
3	3,00	1,00	1,00		
4	4,00	1,00	1,00		
5	1,00	-3,00	-3,00		
6	2,00	1,00	1,00		
7	3,00	1,00	1,00		
8	2,00	-1,00	-1,00		
9	2,00	0,00	0,00		
10	1,00	-1,00	-1,00		
11	2,00	1,00	1,00		
12	3,00	1,00	1,00		
13	3,00	0,00	0,00		
14	3,00	0,00	0,00		
15	5,00	2,00	2,00		
16	5,00	0,00	0,00		
17	4,00	-1,00	-1,00		
18	4,00	0,00	0,00		
19	2,00	-2,00	-2,00		
20	3,00	1,00	1,00		
21	4,00	1,00	1,00		
22	5,00	1,00	1,00		
23	1,00	-4,00	-4,00		
24	2,00	1,00	1,00		
25	3,00	1,00	1,00		
26	4,00	1,00	1,00		

**Figure 6.** Step 2 - SPSS 16 data entry.

By applying the autocorrelation command found at the Analyze → Time Series → Autocorrelations menu of the SPSS 16 software, the output examining the degree of self-correlation of the given series is produced. Consequently, for the given data the result is, see Table 1:

**Table 1.** Level of correlation of the human resources formation estimated on a 1 to 5 scale.

Lag	Autocorrelation	Std. Error <sup>a)</sup>	Box – Ljung Statistic		
			Value	df	Sig. <sup>b)</sup>
1	.343	.070	23.877	1	.000
2	.016	.070	23.926	2	.000
3	.057	.070	24.594	3	.000
4	.296	.070	42.682	4	.000
5	.015	.069	42.729	5	.000
6	-.179	.069	49.396	6	.000
7	-.202	.069	57.927	7	.000
8	-.121	.069	61.004	8	.000
9	-.047	.069	61.465	9	.000
10	-.306	.069	81.356	10	.000
11	-.335	.068	105.312	11	.000
12	-.315	.068	126.683	12	.000
13	-.085	.068	128.259	13	.000
14	-.156	.068	133.573	14	.000
15	-.241	.068	146.221	15	.000
16	-.209	.067	155.770	16	.000

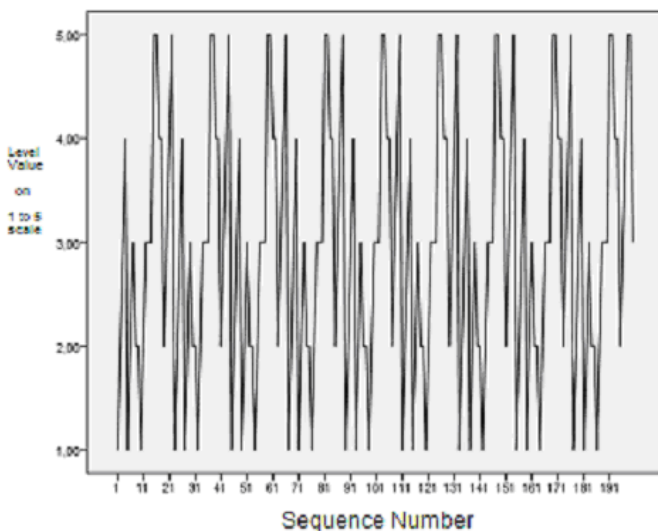
- a) The underlying process assumed is independence (white noise);  
 b) Based on the asymptotic chi-square approximation.

The autocorrelation corresponding to any lag k is calculated following the formula:

$$r_k = \frac{\sum_{t=1}^n X_n(t)X_n(t-k)}{n}$$

In the SPSS 16 software, the autocorrelation test of lags greater than 1 uses the Ljung & Box Test (1979).

**Step 4.** After applying the commands Analyze → Time Series → Sequence Charts, the following chart representing the periodicity of the evolution in time of the dynamic process of continual human resources formation is produced.



**Figure 7.** Chart of human resources formation values estimated on a 1 to 5 scale.

## 4. CONCLUSIONS

The Chaos Theory can be used in the modelling of human resources formation, while the simulation could be carried out by using a SPSS type software.

Using the SPSS software for the given input data, an over time autocorrelation of the continual human resources formation has been established. Consequently, the level of the continual human resources formation at a certain time depends on the previously recorded level and is determined by the repetition interval of the segments displayed in Figure 7.

In conclusion, the evolution of human behavior, the state of skills, knowledge, and attitudes represent a self-correlated temporal series. The fact that the current level of the continual human resources formation depends on the previously recorded level, proves that the evolution of the human resources formation does not have a random character.

## REFERENCES

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