

# MODELLING OF ECONOMIC CYCLES AND MAXIMAL ELEMENTS IN COMPETITIVE ABSTRACT ECONOMIES IN THE CONTEXT OF SUSTAINABLE DEVELOPMENT

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**ABSTRACT:** One of the most frequently debated topics in the current specialized literature is focused on economic cycles [1,5,6,7,8,9,10,18]. Their analysis is highly relevant for long-term predictions, in the attempt to identify solutions for economic increase or for the understanding and solving of economic crises, in view of attaining sustainable development. Their modelling requires the use of differential equations and periodical functions [1]. The aim of the present article is to analyse the properties of these economic cycles, as well as historical aspects regarding mathematical modelling applied to economy. The paper will emphasize that the modelling of economic processes resorts to statistical elements, optimization methods, differential equations, as well as elements of the chaos theory, fixed point theory for multivoice operators; therefore these are elements of modern superior mathematics. Let us illustrate these modellings for the situation of maximal elements in competitive abstract economies.

**Keywords:** economic cycles, modelling, sustainable development, competitive abstract economies, balance, demand, offer

## 1. INTRODUCTION

The cyclicity of the economic process can be defined as its inherent property to evince recurrent values of its status variables, more frequently considered from its dynamic nature, rather than the quantitative value. Its main qualitative determinants are:

- alternation: the economic process subject to cyclicity is alternately undergoing, phenomena of economic increase or decrease;
- periodicity: the economic process subject to cyclicity may qualitatively reverse, after an estimated period of time, to previous values. Mention should be made that the periodicity (frequency) should be estimated not strictly from a calendar perspective, but rather from an economic perspective, over a given period of time, depending on the evolution of certain factors of economic cyclicity;
- inherence: the economic process shall always be subject to cyclicity, as there is no possibility outside this phenomenon;
- cummulative: in the cyclical phenomenon, alternating the economic process trend occurs on the basis of a cummulative process. Throughout this cummulative process, certain functionalities and/or disfunctionalities may reach, in time, specific limits. By reaching these limits, the economic process will move on to the alternating trend;
- Self-adjustment: cyclicity is an economic-cybernetic process characterized by self-adjustment. Alternation of the phases of the economic cycle is a result of the fact that once the economic process has moved to a new phase, all the influence factors start

acting for getting out of this phase. Removal from the phase will take place when the phase will have reached its cumulative limits.

**Thus, cyclicity represents an immanent characteristic of the economic process, and therefore an operating mode of this process.**

## 2. TYPOLOGY OF THE ECONOMIC CYCLE

Specialized literature mentions several classification criteria for the economic cycles, however, duration is the most significant criterion. In this respect, cycles fall into two categories: a) short-term economic cycles (less than a year); b) long-term economic cycles (over a year). Short-term economic cycles are, actually, sub-annual and annual fluctuations of the economic activity. These fluctuations may be:

- weekly fluctuations: triggered by the weekly alternation of some demand-related economic variables: increased or changed expenses over the week-end, weekly salary payment (where applicable) etc.
- monthly fluctuations: triggered by the monthly alternation of some demand-related economic variables in economy: banknote circulation, various consumption;
- seasonal fluctuations: triggered by seasonal alternation, throughout the year, of some demand-related variables of the economic activity: differences in consumption (quantitative and structural) according to season, specific consumption behavior during holiday;
- Annual fluctuations: triggered by annual alternations of some demand variables in economy and they also rely on the fact that the great majority

of economic activities are planned and evaluated annually. Furthermore, most institutional and normative measure (the actual causes of economic cyclicality) are usually taken at the beginning of the year. These annual cycles are more pregnant in the economies where the calendar year coincides with the fiscal year. These fluctuations are also called commercial cycles.

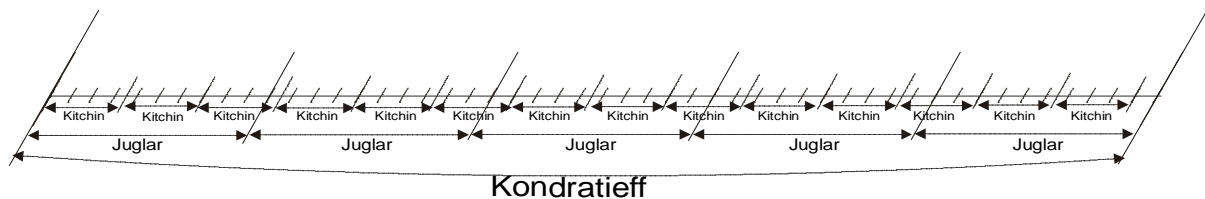
Long-term economic cycles are those cycles that last more than a calendar year.

- Kitchin cycles: they last for about 40 months and they are also called inventory cycles due to the fact that the main reason for producing these cycles is the necessity of renewing stocks of any kind
- Juglar cycles: economic cycles proper, also called ten-year, business or Juglar cycles, after the name of the economist who studied them („Des crises commerciales et leur retour periodique en France, en Angleterre et aux Etats-Unis”, Paris, 1860). Their duration may range from 4-5 years to 10-12 years. They are determined by the significant evolutions of the great industrial processes undergoing in the respective economic system, as

- infrastructural Kuznets cycle (15-25 years) – cyclical variation identified in the structure of aggregate products may be accounted for by prolonged underuse of the production capacities
- Kondratieff cycles: they last for about 50 years and represent the largest descriptive „wave” of alternancies in the economic activities and processes. The Kondratieff cycle (improperly called, sometimes, secular cycle) represents a long-term winding of short-term economic cycles (see [18]).

Therefore, the temporary evolution of an economy should be perceived as the result of various fluctuations occurring simultaneously. The simultaneous existence of long-term economic cycles, with different durations, generates the possibility of their interaction. Hence, genuine economic cycles are affected by a number of alterations of their phases (either in terms of duration or amplitude), according to their place within a winding cycle.

Obviously, Kitchin cycles will be surrounded by Juglar cycles which, in their turn, will be surrounded by Kondratieff cycles (fig. 1):



well as the long-term banking policies.

As we have already mentioned above, this winding has an impact on the shape, amplitude and duration of the cycles phases that are being wound in longer cycles. More specifically, a cycle wound throughout the expansion phase of an winding cycle will evince longer duration and greater amplitude of its expansion phase and, respectively, shorter duration and smaller amplitude in its recession phase. Reversely, a cycle that is being wound in the recession phase of an winding cycle will have an expansion phase with shorter duration and smaller amplitude and, respectively, a recession phase with longer duration and greater amplitude. This is a highly significant result for drawing up long-term policies, especially regarding the restructuring processes or deep economic changes, since it may use the historical „location” where a particular economic system and process are situated, in view of capitalizing specific advantages. Should we resort to a metaphor in this matter we might say that economic systems have to use economic „currents”, just like birds do, in order to maximize their effort and accomplish their target.

### 3. INTEGRATING BUSINESS CYCLES INTO ECONOMIC CYCLES

Business cycles are called endogenous or deterministic, whenever they are generated by the internal mechanism of economy. The term is usually employed to make a clear distinction from traditional models, where fluctuations are triggered by exogenous, most often, random shocks.

There is a great number of studies on empirical methods of testing mechanisms generating non-linear data. Certainly, the theory of business cycles and empirical methods has recorded a long tradition by now. Cycle persistence has been identified by economists as early as the 19th century, whereas thorough theories of fluctuations and business cycles have started to emerge in the fourth and fifth decades of the 20th century, due to the contributions of Frisch (1933), Kalecki (1937), Samuelson (1939), Kaldor (1940), Hicks (1950) and Goodwin (1951).

As early as that time, two potential perspectives regarding the business cycles analysis have emerged: their creation either by oscillation-

generating mechanisms (the so-called non-linear endogenous cycles) or by means of the random-shock impact on a fundamentally stable economic system (the so-called Slutsky-Frisch stochastic cycles).

Before 1970s, the great majority of econometricists focused on the Slutsky-Frisch-type tradition of the business stochastic cycles by means of lineal econometric techniques. They overlooked endogenous cycles and rarely focused on the non-linear data-generating mechanism (except for the studies undertaken by Klein and Preston, 1969).

Studies on non-linearities have rapidly developed recently. It focused on the analysis of univariuous and multivariuous business cycles, and the methods attempted to identify the data-generating mechanisms for oscillating or chaotic movements (Blatt, 1978, Brock, 1986, și Ramsey, 1988).

The new statistical methods helped discover significant non-linear structures in the data series in the fields of economy and finances (Tong, 1990; Brock, Hsie și LeBaron, 1991; Brock, 1992; Granger și Terasvirta, 1993) as well as significant non-linear structure have been modelled. Mention should be made, in Romanian specialized studies, of the studies authored by academician Tudorel Postolache (1981), as early as the 1980s, on the typology and characteristics of cycles approached from a historical perspective.

The first systematic approach to the chronological series regarding business cycles was accomplished by Burns and Mitchell (1946). Most recent studies, however, started from the premise that variables operate according to linear stochastic processes with consistent coefficients. Such an approach allows a better integration of the macroeconomic theory with econometrics. In spite of this integration and a proper understanding of statistical properties, compared to the „richness” of Burns-Mitchell analysis, the potential analysis of asymmetries between recessions and expansions or the temporal notion of business cycles (as opposed to calendar cycles) have disappeared.

The major problem of macroeconomists as regards the fluctuation description resided in the distinction of the trend from the cyclic component. Although this separation may be interpreted from a purely statistical perspective, many economists still believe that, given the absence of short-term fluctuations, the economy will evince an overall increasing trend which might be considered as the one aimed at.

Economic historians have also identified some distinct cycles such as the short-term business cycle (3-7 years), the construction or Kuznets cycle (15-25 years), as well as the long-wave of Kondratieff cycle (40-60 years). There is the natural question, therefore, how can different cycles interact?

First and foremost, economic systems differ from most other systems in the natural sciences due to the prevailing character of positive reverse reaction loops. Best known examples in this respect refer to the accelerator and the Keynes theory of multiplying loops.

Other positive loops operate in the case of the so-called extrapolative expectations, crowding-out effect, increased efficiency, the effect of inflationist expectations on the real interest rates and therefore the aggregate demands, the financial crises and speculations as well as the synergy among and within the technologies employed for production, communications and organizations (Graham and Senge, 1980, Sterman, 1986, Arthur, 1988, Semmler, 1989, a.s.o.).

It is obvious thus that the development of non-linear concepts and methods represents a product of the computer era. Most studies in the initial stages started from the numerical analysis of extremely simple non-linear models, what nowadays are merely the fundamentals of data processing and PC calculation. The conclusion reached was that even the simplest non-linear models are able to reproduce a wide variety of characteristics.

For instance, one conclusion was that small changes in the parameter values lead to surprising results, such as simple classical models, previously considered as having a cyclical and even easily predictable behaviour. This has entailed the idea of „determinist chaos”. Another significant feature of logistics, whenever employing its representation as bifurcation diagram, is that every time a window is closely looked at (higher resolution), the diagram is represented as an identical copy of the integral diagram.

The analysis highlighted the great diversity of characteristics of an apparently simple model, where the standard idea of a static balance, intensively employed in linear economic models, can describe only the very limited area of possibilities of a monotonous behaviour or with regular oscillations.

For the past few years, the discoveries regarding the theory of non-linear dynamic systems have generated significant changes in the classical theory of economics. The for non-linear and deterministic

systems has continuously intensified and related studies have developed.

There are at least two reasons that make chaos theory so appealing in order to explain the economic phenomenon. The former is the interest evinced by scientists for creating realistic models. Most characteristic of the chaos theory are obviously in contradiction with fundamental notions of classical economy, the very foundation of contemporary economic theory.

The classical theory defines the universe as a mechanism that might be subject to accurate measurements, precise predictions and control. Chaos, on the other hand, is a deterministic system characterized by complex behaviour, based on apparently hazardous interaction among elements.

The latter is the possibility of controlling systems. The initial opinion upheld by the scientists was that the chaotic movement of systems is neither predictable nor controllable because of the sensitive dependence to initial circumstances. Small changes lead to other chaotic movements, not to any stable or predictable alternative.

Scientists managed to demonstrate, however, that chaos may be unpredictable but its deterministic structure makes it exploitable and consequently controllable. Paradoxically, the sensitivity of the chaotic system to initial circumstances may be used to control it. This observation has opened up the possibility of changing the behavior of natural systems without altering any of the initial characteristics. Nevertheless, chaos theory has prompted economists to study the following contradictions to classical theory: economics is a non-linear science, its dynamics leads to multiple solutions, and economic units are non-representatives. Compared to the models of neo-classical theory, the application of chaos theory to economics enables researchers to interpret phenomena previously considered stochastic, exogenous and unlikely to be influenced [5,6,7,8].

#### 4. COMPETITIVE ABSTRACT ECONOMIES

The aim of this section is to highlight some of the most significant economic models. Let us mention some mathematical results, especially those related to the analysis of multivocal operators, since the results have particular relevance in the context of various economic problems.

The Arrow-Debreu model (1959) has been designed for an economy considered to have a finite number of companies. This is one of the most significant studies in the modern theory of general balance and

it refers to a Walrasian general balance. The economic interpretation of the Walras law is that in a closed economy almost the entire individual income will be spent. There are two illustrations of the Walrasian balance in specialized studies. The former uses the classical notion of request and offer functions. A price vector is considered to be balanced provided that the request equals the offer. The latter is independent from the request and offer functions.

In the classical Arrow-Debreu model, a finite number of goods are traded, produced or consumed. As regards merchandise such as: steel, cereal or apples, these are available for various time intervals, locations or states. Let us consider there are  $l$  such merchandise. Then it is normal to look at the  $E$  area of merchandise as a sub-multitude of  $R^l$ . The consumers' preference more instead of less has significant consequences for the balance analysis. On the other hand, the fundamental principle of theoretical economy is that agents behave rationally, i.e. they are aware of their own interest and their actions are aimed at maximizing profit [2,11].

An  $x$  vector from  $E$  includes on each component the product quantity that is subject to a specific transaction at a given time. Therefore, it is not individual products but rather these vector-merchandise that will be produced, traded or consumed. Of course, some of the  $x$  components may be null.

Let  $p$  stand for the price vector, i.e. the vector that includes on each component the value of a product unit. The value of  $x$  may then be calculated as a scalar product as follows:

$$(p, x) = \sum_{i=1}^l p_i x_i \quad (1)$$

The working hypothesis is a finite number of consumers.

Certainly, not every merchandise vector is available as final consumption for a consumer. Therefore  $x_i \subseteq R^l$  stands for the multitude of all merchandise vector available to the  $i$  consumer, i.e. its consumption multitude. Let us remember that the admissible merchandise vector are positive.

Let us consider  $A_i$  the budget multitude of the  $i$  consumer, i.e. the multitude of those merchandise vectors of the consumption multitude available and affordable at a  $p$  price and  $M_i$  budget.

$$A_i = \{x \in X | (p, x) \leq M_i\} \quad (2)$$

Certainly the budget multitude may also be void. In the context of a market economy, any consumer is confronted with the problem of selecting one or

more elements from the  $A_i$  budget multitude. The choice of one or another of the elements of this multitude shall be made according to a given criterion. One method of defining such a criterion requires the use and existence of a utility function  $u_i: X_i \rightarrow R$ . The  $i$  consumer will choose the  $x$  over  $y$  merchandise vector provided that  $u_i(x) > u_i(y)$  and it makes no difference which one to choose if  $u_i(x) = u_i(y)$ . Therefore, the mathematical problem of the consumer is to maximize the utility function on the  $A_i$  budget multitude.

Mathematical analysis shows that if the utility function is continuous and the  $A_i$  multitude is compact, on the basis of the Weierstrass theorem, then there are  $x$  vectors that maximize the  $u_i$  function on the  $A_i$  multitude. However, economic practice shows that such constrains are too harsh and there are lighter hypotheses of consumer preferences that enable the problem solution. The description of  $i$  consumer preference will be accomplished by means of the multivocal operator of preferences  $U_i: A_i \rightarrow P(A_i)$ . For instance,

$$U_i(x) = \{y | u_i(y) > u_i(x)\} \\ = \{y | y \text{ este strict preferat}\}$$

will be selected. It is obvious that  $x^* \in A_i$  is the optimum preference if  $U_i(x^*)$  is void. Such an element is called  $U$  – maximal. One can easily demonstrate that any fixed point theorem for multivocal operator generates an existence result of a  $U$  - maximal point for the preference function [13,15,16].

## 5. SUSTAINABLE DEVELOPMENT BY EFFECTIVE ECONOMIC ACTIVITIES. CONCLUSIONS

One of the methods of sustainable development is to make economic activities more effective, considering the urgency of minimizing production costs, thus entailing a complex capitalization of raw materials, minimizing the consumption raw materials well as oil and energy [12].

The market economy is performative by excellence, due to the fact that it is free and competitive. Given the lack of competitiveness, the product quality will diminish accompanied, paradoxically, by an increase in the price. Thus the „amazing” result is poor quality products available on the market at excessive prices, associated with very good „sales” – because of their low reliability, products (parts) have to be replaced very often in the production processes.

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